# A Cost Model for Path Expressions in Object-Oriented Queries 

Cetin Ozkan Asuman Dogac Mehmet Altinel<br>Software Research and Development Center<br>Scientific and Technical Research Council of Turkiye<br>Middle East Technical University<br>06531, Ankara Turkiye<br>e-mail: asuman@vm.cc.metu.edu.tr


#### Abstract

Query processing remains one of the important challenges of Object-Oriented Database Management Systems. Cost based query optimization involves creating alternative executing plans for a given query and executing the least costly one within a cost model framework.

In Object-Oriented Database Management Systems (OODBMSs) objects may store references to other objects (precomputed joins), and path expressions are used in query languages. Although the cost fomulas for explicit joins and the selectivities of attributes and joins are well-known in the relational model, there is no similar work involving path expressions for OODBMSs. However in order to optimize object-oriented queries involving path expressions, a cost model is essential. This information is necessary for deciding whether to use pointer chasing or to convert the path expressions into explicit joins and also for deciding the execution order of path expressions. In this paper, we provide a cost model that includes the formulas for the costs and selectivities of forward and backward path traversals.


## 1. Introduction

The goal of query optimization is to find an execution plan for a specific query in order to minimize the cost of executing the query. The steps involved in this process can be considered at two levels, logical query optimization (query rewriting) using semantic properties of the language in order to find expressions equivalent to the one given by the user, and physical query optimization, based on a cost model to choose the best algorithm for evaluating the query.

In calculating the cost of an execution plan for object-oriented queries, estimation of the cost of forward and backward traversals in path expressions is necessary. Although executing precomputed joins is not the best in all situations, estimations of the cost of path traversals is essential to compare their costs with other possible alternatives to choose the best performing access plan. The fact that path traversals must be taken into account when deciding on an execution plan is demonstrated through the following query:

SELECT emp.name, emp.job.name, emp.dept.name
FROM Employee emp
WHERE emp.dept.plant.location='Dallas' and emp.eno=15
Assuming that there is no index in any of the extensions of the classes, this query will require one sequential scan of the extension of the Employee class and 3 disk accesses, (one to fetch the corresponding dept object, one to retrieve the plant object and the last one to fetch the job object) when the path expression is evaluated through pointer chasing. If this query is executed with an explicit join technique it will require the sequential scan of the extensions of the emp, dept, plant, and job classes, and even for very small sized extensions, this cost will exceed the cost of path traversal.

On the other hand, the optimal execution plan for the following query is through explicit joins [Bla 93] :

SELECT emp.name, emp.job.name, emp.dept.name
FROM Employee emp
WHERE emp.dept.plant.location='Dallas'
It is clear that to be able to decide on the better execution plan, we should be able to calculate the cost of possible alternatives. Although the cost formulas for explicit joins and selectivities of attributes, and joins are well-known in relational model, there is no similar work in calculating the selectivities and costs of path expressions for OODBMSs.

In this paper, we provide the formulas for calculating the selectivities of path expressions, for estimating the size of an explicit join involving path expressions, and for calculating the costs of forward and backward traversals.

Previous work on object-oriented query optimization involved the definition of new object algebras [Alh 93, Sha 90, Str 90], query rewriting techniques [Clu 92], and new execution algorithms to efficiently traverse complex object structures such as pointer based joins [She 90] and complex object assembly [Kel 91]. Recently the design and implementation of a query optimizer based on complete extensible framework has been reported in [Bla 93]. In [Bla 93], only the selectivities of the attributes are considered when there is index on them that can be used to assist selectivity estimation. In other cases, selectivities are assumed to be $10 \%$.

## 2. Cost Model Parameters

In the object model [Atk 92] used in this paper, complex objects are built from simpler ones by applying constructors to them. The simplest objects are integers, characters, byte strings of any length, Booleans, and floats. The complex object constructors are Tuple, Set, List and Reference. Any constructor can be applied to any object. Each object has a unique Object Identifier (OID). Objects are grouped in the abstraction level of a class, in other words, classes have extensions. Each object is as a member of only one class.

In this section cost model parameters are defined and calculated for the object model presented. These parameters are used in various selectivity calculations which form the basis of the cost calculation of path traversals. In the Table 1, the cost model parameters are presented. Similar cost model parameters have been defined in [Kem 90], [Ber 92] and [Ber 93]. In defining the cost model, [Kem 90] considers the extensions of classes where in [Ber 92] and [Ber 93], the class inheritance hierarchy is also taken into account. Our cost model considers the class extensions. Furthermore we have defined some more parameters that serve our purposes better.

In the Table $1, C_{i}$ is a class, $A$ is either an attribute or a parameterless method of class $C_{i}$ with an atomic return type which is treated in the same way as an atomic attribute, and $\mathrm{C}_{\mathrm{i}+1}$ is the class referenced by attribute $A$ of class $C_{i}$.

| Parameter | Short Hand Notation | Definition |
| :---: | :---: | :---: |
| \| $\mathrm{C}_{\mathrm{i}}$ \| | - | Total number of instances of $\mathrm{C}_{\mathrm{i}}$ |
| nbpages( $\mathrm{C}_{\mathrm{i}}$ ) | - | Total number of pages $\mathrm{C}_{\mathrm{i}}$ occupies |
| $\operatorname{size}\left(\mathrm{C}_{\mathrm{i}}\right)$ | - | Size of an instance of class $\mathrm{C}_{\mathrm{i}}$ |
| notnull( $\mathrm{A}, \mathrm{C}_{\mathrm{i}}$ ) | - | The proportion of instances of class $\mathrm{C}_{\mathrm{i}}$ where attribute A is not null |
| $\operatorname{fan}\left(\mathrm{A}, \mathrm{C}_{\mathrm{i}}, \mathrm{C}_{\mathrm{i}+1}\right)$ | $\mathrm{fan}_{\mathrm{i}}(\mathrm{A})$ | The average number of instances of class $\mathrm{C}_{\mathrm{i}+1}$ that are referenced by an instance of $\mathrm{C}_{\mathrm{i}}$ through attribute A |
| totref( $\left.\mathrm{A}, \mathrm{C}_{\mathrm{i}}, \mathrm{C}_{\mathrm{i}+1}\right)$ | totref $_{\text {i }}(\mathrm{A})$ | The total number of objects in class $\mathrm{C}_{\mathrm{i}+1}$ that are referenced by at least one object in class $\mathrm{C}_{\mathrm{i}}$ through attribute A |
| $\operatorname{dist}\left(\mathrm{A}, \mathrm{C}_{\mathrm{i}}\right)$ | $\operatorname{dist}_{\mathrm{i}}(\mathrm{A})$ | The number of distinct values of the atomic attribute A of class $\mathrm{C}_{\mathrm{i}}$ |
| $\max \left(\mathrm{A}, \mathrm{C}_{\mathrm{i}}\right)$ | $\max _{\mathrm{i}}(\mathrm{A})$ | The maximum value of the atomic attribute A of class $\mathrm{C}_{\mathrm{i}}$ |
| $\min \left(\mathrm{A}, \mathrm{C}_{\mathrm{i}}\right)$ | $\min _{i}(\mathrm{~A})$ | The mimimum value of the atomic attribute A of class $\mathrm{C}_{\mathrm{i}}$ |

Table 1. Cost Model Parameters
The parameters calculated by using the above listed cost model parameters are as follows: The number of the total references from class $C_{i}$ to class $C_{i+1}$ through attribute $A$ is denoted by totlinks $\left(\mathrm{A}, \mathrm{C}_{\mathrm{i}}, \mathrm{C}_{\mathrm{i}+1}\right)$ and given by the following equation :

$$
\operatorname{totlinks}\left(\mathrm{A}, \mathrm{C}_{\mathrm{i}}, \mathrm{C}_{\mathrm{i}+1}\right)=\operatorname{fan}\left(\mathrm{A}, \mathrm{C}_{\mathrm{i}}, \mathrm{C}_{\mathrm{i}+1}\right) *\left|\mathrm{C}_{\mathrm{i}}\right|
$$

The probability that an instance of class $\mathrm{C}_{\mathrm{i}+1}$ is referenced by the instances of class $\mathrm{C}_{\mathrm{i}}$ through attribute A is given by the following formula:

$$
\operatorname{hitprb}\left(\mathrm{A}, \mathrm{C}_{\mathrm{i}}, \mathrm{C}_{\mathrm{i}+1}\right)=\operatorname{totref}\left(\mathrm{A}, \mathrm{C}_{\mathrm{i}}, \mathrm{C}_{\mathrm{i}+1}\right) /\left|\mathrm{C}_{\mathrm{i}+1}\right|
$$

The shorthand notation for these parameters as follows:

$$
\begin{aligned}
& \operatorname{hitprb}_{\mathrm{i}}(\mathrm{~A})=\operatorname{hitprb}\left(\mathrm{A}, \mathrm{C}_{\mathrm{i}}, \mathrm{C}_{\mathrm{i}+1}\right) \\
& \operatorname{totlinks}_{\mathrm{i}}(\mathrm{~A})=\operatorname{totlinks}\left(\mathrm{A}, \mathrm{C}_{\mathrm{i}}, \mathrm{C}_{\mathrm{i}+1}\right)
\end{aligned}
$$

### 2.1 Selectivity

Selectivity is a parameter used with a predicate to denote the ratio of the elements of a collection satisfying the predicate. When optimizing the queries, selectivity of a predicate is estimated assuming that the values are uniformly distributed. The traditional uniformity and randomness assumptions about value distributions and object placements tend to overestimate costs. However the more sophisticated techniques require more statistical information about the database. The question of how to maintain such information within tolerable overhead is not yet fully resolved [Jar85].

A simple predicate in the system is a triplet of the form $<\mathrm{P}, \Theta$, oprnd>, where P is a path expression, $\Theta$ is a comparison operator ( $=,\langle>,>=,\langle=,>,<$ ), and oprnd is either a constant or another path expression.

### 2.1.1 Selectivity for Atomic Attributes

The well-known selectivity calculations described in [Ozk 90] that assume a uniform distribution of the atomic values will be used throughout the derivations presented in this paper.
i. The selectivity of the predicate <p.A, $=$,constant> denoted $\mathrm{f}_{\mathrm{s}}$ (p.A, $=$ ), where p is a variable bound to a class $\mathrm{C}_{\mathrm{i}}$, and A is an atomic attribute, is given by the following formula.

$$
\mathrm{f}_{\mathrm{s}}(\mathrm{p} . \mathrm{A},=)=1 / \operatorname{dist}_{\mathrm{i}}(\mathrm{~A})
$$

ii. The selectivity of the predicate < p.A, > ,constant> is

$$
\mathrm{f}_{\mathrm{s}}(\mathrm{p} . \mathrm{A},>)=\left(\max _{\mathrm{i}}(\mathrm{~A})-\text { constant }\right) /\left(\max _{\mathrm{i}}(\mathrm{~A})-\min _{\mathrm{i}}(\mathrm{~A})\right)
$$

iii. The selectivity of the predicate <p.A, BETWEEN, constant ${ }_{1}$ and constant ${ }_{2}>$, where constant ${ }_{1}$ is less than constant ${ }_{2}$, is

$$
\mathrm{f}_{\mathrm{s}}(\mathrm{p} . \mathrm{A}, \text { BETWEEN })=\left(\text { constant }_{2}-\operatorname{constant}_{1}\right) /\left(\max _{i}(\mathrm{~A})-\min _{i}(\mathrm{~A})\right)
$$

The parameterless methods of a class $\mathrm{C}_{\mathrm{i}}$ with atomic return types are treated in the same manner
as the atomic data members of that class.

### 2.1.2. Selectivity of Path Expressions

Assume that there is a path expression that contains $m$ attributes, $A_{1}$ through $A_{m}$, where $A_{1}$ through $A_{m-1}$ is constructed using the set and the reference constructors. $A_{m}$ is an atomic attribute and $A_{i}$ is an attribute of class $C_{i}$. We define the selectivity, $f_{s}\left(p . A_{1} \cdot A_{2} \ldots A_{m}, \Theta\right)$, for the path expression predicate "p. $A_{1} \cdot A_{2} \ldots A_{m} \Theta c$ ", where $\Theta$ is a comparison operator and $c$ is a constant, as the ratio of the elements of the starting collection (to which p is bounded) satisfying the predicate.

The calculation of the selectivity of the predicate $\left\langle p . A_{m}, \Theta, c\right\rangle, f_{s}\left(p . A_{m}, \Theta\right)$, is clear from the previous section. Therefore the expected number of instances of $C_{m}$, denoted by $k_{m}$, that satisfies this condition is:

$$
\mathrm{k}_{\mathrm{m}}=\left|\mathrm{C}_{\mathrm{m}}\right| * \mathrm{f}_{\mathrm{s}}\left(\mathrm{p} . \mathrm{A}_{\mathrm{m}}, \Theta\right)
$$

Consider the following problem: When we start with $k$ objects in class $C_{1}$ and follow $k * \operatorname{fan}_{1}\left(\mathrm{~A}_{1}\right)$ links through attribute $A_{1}$ into class $\mathrm{C}_{2}$, how many objects do we obtain in class $\mathrm{C}_{2}$ ? Note that the total number of objects referenced in class $C_{2}$ through attribute $A_{1}$ of class $C_{1}$ is totref $\left(A_{1}\right)$. This question can be reduced to the following statistical problem: Given $n$ objects uniformly distributed over m colors, how many different colors c are expected to be selected if we take r objects? This statistical problem has been solved by using different mathematical approximations. An approximation assumed in [Cer 85] is as follows:

$$
\mathrm{c}(\mathrm{n}, \mathrm{~m}, \mathrm{r})=\begin{array}{ll}
\mathrm{r} & , \mathrm{r}<\mathrm{m} / 2 \\
(\mathrm{r}+\mathrm{m}) / 3 & , \mathrm{~m} / 2 \leq \mathrm{r} \leq 2 \mathrm{~m} \\
\mathrm{~m} & , \mathrm{r}>2 \mathrm{~m}
\end{array}
$$

When we generalize this assuming that we start with k objects of class $\mathrm{C}_{1}$ and traverse the path p. $A_{1} . A_{2} \ldots A_{i}$ in forward direction, the expected number of objects of class $C_{i+1}$, denoted by fref, is given by the following formula:
$\begin{array}{ll}\operatorname{fref}\left(\text { p. } \cdot A_{1} \cdot A_{2} \ldots A_{i}, k\right)= & , i=0 \\ c\left(\text { totlinks }_{i}\left(A_{i}\right), \operatorname{totref}_{i}\left(A_{i}\right), \operatorname{fref}\left(\text { p. } A_{1} \cdot A_{2} \ldots A_{i-1}, k\right) * \operatorname{fan}_{i}\left(A_{i}\right)\right), \quad i>0\end{array}$
Note that [Car 75] and [Yao 77], in approximating the number of block accesses to a file for a given query, has defined better approximations to this statistical problem. However, we have calculated and compared the average error per unit time, and found out that $\mathrm{c}(\mathrm{n}, \mathrm{m}, \mathrm{r})$ serves our purposes well.

Starting with one instance of class $C_{1}$, the number of objects of class $C_{m}$ obtained at the end of forward path traversal is given by fref(p. $\left.\mathrm{A}_{1} \ldots \mathrm{~A}_{\mathrm{m}-1}, 1\right)$ which will be denoted as set $\mathrm{S}_{1}$. On the other hand, there are $k_{m}$ objects satisfying the predicate $\left\langle\right.$ p. $\left.A_{m}, \Theta, c\right\rangle$ in the extent of $C_{m}$ and this set will
be denoted by $\mathrm{S}_{2}$. The number of objects that are referenced by the objects in the class $\mathrm{C}_{\mathrm{m}-1}$ in $\mathrm{S}_{2}$ can be calculated as $\mathrm{k}_{\mathrm{m}}{ }^{*}$ hitprb $\left(\mathrm{A}_{\mathrm{m}-1}, \mathrm{C}_{\mathrm{m}-1}, \mathrm{C}_{\mathrm{m}}\right)$. Denoting this set as $\mathrm{S}_{3}$, Figure 1 illustrates the set containment relationship among $S_{1}, S_{2}$ and $S_{3}$ assuming that $S_{1}$ intersects with $S_{2}$ and $S_{3}$ although this is not the case always.


Figure 1 Relation between $S_{1}, S_{2}$ and $S_{3}$
It is clear that if the sets $S_{1}$ and $S_{3}$ intersects then the object we have started with in $C_{1}$ satisfies the predicate $\left\langle\mathrm{p} . \mathrm{A}_{1} \ldots \mathrm{~A}_{\mathrm{m}}, \Theta, \mathrm{c}\right\rangle$. Therefore, the selectivity of a path expression can be defined as the probability that the sets $S_{1}$ and $S_{3}$ will intersect. This probality, $P_{\text {intersect }}$, can be calculated as follows:

$$
P_{\text {intersect }}=o\left(\text { totref }_{m-1}\left(A_{m-1}\right), \operatorname{fref}\left(\text { p. } A_{1} \cdot A_{2} \ldots A_{m-1}, 1\right), k_{m} * \operatorname{hitprb}\left(A_{m-1}, C_{m-1}, C_{m}\right)\right)
$$

where $o(t, x, y)$ is the probability that there exists at least one object in common in two sets having cardinalities x and y respectively ( $\mathrm{x}, \mathrm{y} \leq \mathrm{t}$ ). Elements of these sets are selected without replacement from the same set of distinct objects seperately (i.e. after the construction of first set, its elements are placed back to the original set for the selection of the second set). $o(t, x, y)$ is defined as:

$$
\begin{array}{ll}
1 & , \mathrm{x}+\mathrm{y}>\mathrm{t} \\
\mathrm{o}(\mathrm{t}, \mathrm{x}, \mathrm{y})= & 1-\mathrm{C}(\mathrm{t}-\mathrm{x}, \mathrm{y}) / \mathrm{C}(\mathrm{t}, \mathrm{y})
\end{array}
$$

where $C(u, v)$ stands for the number of combinations of $u$ objects selected from a set of $v$ objects without replacement.

Thus the selectivity of a path expression $f_{s}\left(\right.$ p. $\left.A_{1} \cdot A_{2} \ldots A_{m}, \Theta\right)$ is:

$$
\mathrm{f}_{\mathrm{s}}\left(\mathrm{p} \cdot \mathrm{~A}_{1} \cdot \mathrm{~A}_{2} \ldots \mathrm{~A}_{\mathrm{m}}, \Theta\right)=\mathrm{P}_{\text {intersect }}
$$

### 2.1.3. Estimating the Size of a Join

Assume that a join predicate is defined on two path expressions as follows:

$$
\text { p. } A_{1} \cdot A_{2} \ldots A_{m}=s . B_{1} \cdot B_{2} \ldots B_{n} .
$$

$A_{i}$ is an attribute of $C_{i}$, and $B_{j}$ is an attribute of $D_{j}$. The number of instances of $C_{1}$ and $D_{1}$ are denoted by $t_{p}$ and $t_{s}$, respectively. Notice that if there are no previous selections on these classes, then $\mathrm{t}_{\mathrm{p}}=\left|\mathrm{C}_{1}\right|$ and $\mathrm{t}_{\mathrm{s}}=\left|\mathrm{D}_{1}\right|$.

Estimation of the size of a join size necessary for the optimizer to make correct estimates of the costs of alternative query execution plans. The join size will be estimated for three cases.

## Case 1. Object Identifier Equality

Our cost model considers the class extensions and an object can be an instance of only one class. Therefore when we consider the object identifier equality, it is clear that $A_{m}$ and $B_{n}$ reference the same class, say E . For one instance of $\mathrm{C}_{1}$, the expected number of instances of E obtained by traversing the path expression starting with the bind variable $p$ is

$$
\mathrm{k}_{\mathrm{p}}=\operatorname{fref}\left(\mathrm{p} \cdot \mathrm{~A}_{1} \cdot \mathrm{~A}_{2} \ldots \mathrm{~A}_{\mathrm{m}}, 1\right)
$$

In the same way,

$$
\mathrm{k}_{\mathrm{s}}=\operatorname{fref}\left(\mathrm{s} \cdot \mathrm{~B}_{1} \cdot \mathrm{~B}_{2} \ldots \mathrm{~B}_{\mathrm{n}}, 1\right)
$$

We will define the selectivity of a join predicate defined on two path expressions:

$$
\text { p. } A_{1} \cdot A_{2} \ldots A_{m}=s \cdot B_{1} \cdot B_{2} \ldots B_{n} \text {, where both } A_{m} \text { and } B_{n} \text { contain object identifiers. }
$$

Starting with one instance of class $\mathrm{C}_{1}$, the number of objects of class $E$ obtained at the end of forward path traversal is given by $\mathrm{k}_{\mathrm{p}}$. However not all of these objects may be hit by the links coming from class $D_{n}$. Thus $k_{p} * \operatorname{hitprb}\left(B_{n}, D_{n}, E\right)$ gives the size of the set whose elements are also hit by the links coming from class $D_{n}$. Similarly $k_{s} * \operatorname{hitprb}\left(A_{m}, C_{m}, E\right)$ gives the size of the set which is obtained by starting with one instance of class $D_{1}$ and following the path s. $B_{1} \cdot B_{2} \ldots B_{n}$ and whose elements are also hit by the links coming from class $\mathrm{C}_{\mathrm{m}}$. Notice that both of these sets are contained in the set whose cardinality is totref $\left(\mathrm{A}_{\mathrm{m}}, \mathrm{C}_{\mathrm{m}}, \mathrm{E}\right) *$ totref $\left(\mathrm{B}_{\mathrm{n}}, \mathrm{D}_{\mathrm{n}}, \mathrm{E}\right) /[\mathrm{E} \mid$. Notice also that totref $\left(B_{n}, D_{n}, E\right) /\left[E \mid\right.$ is hitprb $\left(B_{n}, D_{n}, E\right)$. The probability, $P_{i}$, that these two sets intersects is given by the following formula:

$$
\mathrm{P}_{\mathrm{i}}=\mathrm{o}\left(\operatorname{totref}\left(\mathrm{~A}_{\mathrm{m}}, \mathrm{C}_{\mathrm{m}}, \mathrm{E}\right) * \operatorname{hitprb}\left(\mathrm{~B}_{\mathrm{n}}, \mathrm{D}_{\mathrm{n}}, \mathrm{E}\right), \mathrm{k}_{\mathrm{p}}^{*} * \operatorname{hitprb}\left(\mathrm{~B}_{\mathrm{n}}, \mathrm{D}_{\mathrm{n}}, \mathrm{E}\right), \mathrm{k}_{\mathrm{s}} * \operatorname{hitprb}\left(\mathrm{~A}_{\mathrm{m}}, \mathrm{C}_{\mathrm{m}}, \mathrm{E}\right)\right) .
$$

We define the selectivity, q , of a join predicate defined on two path expressions p. $\mathrm{A}_{1} \cdot \mathrm{~A}_{2} \ldots \mathrm{~A}_{\mathrm{m}}=$ s. $B_{1} \cdot B_{2} \ldots B_{n}$ as the probability of an object being in the intersection of two sets with cardinalities
$\mathrm{k}_{\mathrm{p}} * \operatorname{hitprb}\left(\mathrm{~B}_{\mathrm{n}}, \mathrm{D}_{\mathrm{n}}, \mathrm{E}\right)$ and $\mathrm{k}_{\mathrm{s}} * \operatorname{hitprb}\left(\mathrm{~A}_{\mathrm{m}}, \mathrm{C}_{\mathrm{m}}, \mathrm{E}\right)$ selected from a set whose cardinality is totref $\left(\mathrm{A}_{\mathrm{m}}, \mathrm{C}_{\mathrm{m}}, \mathrm{E}\right) *$ hitprb $\left(\mathrm{B}_{\mathrm{n}}, \mathrm{D}_{\mathrm{n}}, \mathrm{E}\right)$ with replacement.

$$
\mathrm{q}=\mathrm{P}_{\mathrm{i}} *\left(\mathrm{k}_{\mathrm{p}} * \operatorname{hitprb}\left(\mathrm{~B}_{\mathrm{n}}, \mathrm{D}_{\mathrm{n}}, \mathrm{E}\right) * \mathrm{k}_{\mathrm{s}} * \operatorname{hitprb}\left(\mathrm{~A}_{\mathrm{m}}, \mathrm{C}_{\mathrm{m}}, \mathrm{E}\right) /\left(\operatorname{totref}\left(\mathrm{A}_{\mathrm{m}}, \mathrm{C}_{\mathrm{m}}, \mathrm{E}\right) * \operatorname{hitprb}\left(\mathrm{~B}_{\mathrm{n}}, \mathrm{D}_{\mathrm{n}}, \mathrm{E}\right)\right)^{* *} 2\right) .
$$

The size of the resulting join, hence, is calculated to be:

$$
\text { join_size }=\operatorname{fref}\left(\mathrm{p} \cdot \mathrm{~A}_{1} \cdot \mathrm{~A}_{2} \ldots \mathrm{~A}_{\mathrm{m}}, \mathrm{t}_{\mathrm{p}}\right) * \operatorname{fref}\left(\mathrm{~s} \cdot \mathrm{~B}_{1} \cdot \mathrm{~B}_{2} \ldots \mathrm{~B}_{\mathrm{n}}, \mathrm{t}_{\mathrm{s}}\right) * \mathrm{q} .
$$

Case 2. Value-based Equality
After forward traversing the path p. $\mathrm{A}_{1} \cdot \mathrm{~A}_{2} \ldots \mathrm{~A}_{\mathrm{m}-1}$ starting with one instance of class $\mathrm{C}_{1}$, we have obtained $k_{p}$ objects in class $C_{m}$. We need to find out the number of non-null distinct values of attribute $A_{m}$ corresponding to $k_{p}$ objects. There is a total of $\left|C_{m}\right| *$ notnull $\left(A_{m}, C_{m}\right)$ number of nonnull values of $A_{m}$ distributed over $\operatorname{dist}\left(\mathrm{A}_{\mathrm{m}}, \mathrm{C}_{\mathrm{m}}\right)$ number of distinct values. When we select $\mathrm{k}_{\mathrm{p}}{ }^{*}$ notnull $\left(A_{m}, C_{m}\right)$ number of values out of $\left|C_{m}\right| *$ notnull $\left(A_{m}, C_{m}\right)$ values, the number of non-null distinct values of attribute $A_{m}$ corresponding to $k_{p}$ objects, denoted by $k_{p}{ }^{\prime}$ is:

$$
\mathrm{k}_{\mathrm{p}}^{\prime}=\mathrm{c}\left(\left|\mathrm{C}_{\mathrm{m}}\right| * \operatorname{notnull}\left(\mathrm{~A}_{\mathrm{m}}, \mathrm{C}_{\mathrm{m}}\right), \operatorname{dist}\left(\mathrm{A}_{\mathrm{m}}, \mathrm{C}_{\mathrm{m}}\right), \mathrm{k}_{\mathrm{p}}^{*} \operatorname{notnull}\left(\mathrm{~A}_{\mathrm{m}}, \mathrm{C}_{\mathrm{m}}\right)\right) .
$$

and similarly,

$$
\mathrm{k}_{\mathrm{s}}^{\prime}=\mathrm{c}\left(\left|\mathrm{D}_{\mathrm{n}}\right| * \operatorname{notnull}\left(\mathrm{~B}_{\mathrm{n}}, \mathrm{D}_{\mathrm{n}}\right), \operatorname{dist}\left(\mathrm{B}_{\mathrm{n}}, \mathrm{D}_{\mathrm{n}}\right), \mathrm{k}_{\mathrm{s}} * \operatorname{notnull}\left(\mathrm{~B}_{\mathrm{n}}, \mathrm{D}_{\mathrm{n}}\right)\right) .
$$

Then the range of the two-path value-based join is defined to be

$$
\operatorname{range}_{\text {join }}=\left[\max \left(\min \left(A_{m}, C_{m}\right), \min \left(B_{n}, D_{n}\right)\right), \min \left(\max \left(A_{m}, C_{m}\right), \max \left(B_{n}, D_{n}\right)\right)\right]
$$

On the other hand, range of each path expression is as follows:

```
range }\mp@subsup{p}{p}{}=[\operatorname{min}(\mp@subsup{A}{m}{},\mp@subsup{C}{m}{}),\operatorname{max}(\mp@subsup{A}{m}{m},\mp@subsup{C}{m}{})]
range }=[\operatorname{min}(\mp@subsup{B}{n}{},\mp@subsup{D}{n}{}),\operatorname{max}(\mp@subsup{B}{n}{},\mp@subsup{D}{n}{})]
```

Number of distinct values of $\mathrm{k}_{\mathrm{p}}$ objects which fall into the range $\mathrm{j}_{\mathrm{join}}$, denoted by Dist $_{\mathrm{p}}$ is calculated as:
$\operatorname{Dist}_{\mathrm{p}}=\mathrm{k}_{\mathrm{p}}^{\prime} * \operatorname{len}\left(\right.$ range $\left._{\mathrm{join}}\right) / \operatorname{len}\left(\right.$ range $\left._{\mathrm{p}}\right)$, where $\operatorname{len}($ range $)=\mathrm{i}-\mathrm{j}$, and range $=[\mathrm{j}, \mathrm{i}]$.
Similarly,
Dist $_{\mathrm{s}}=\mathrm{k}_{\mathrm{s}}{ }^{\prime} *$ len $\left(\right.$ range $\left._{\text {join }}\right) /$ len $\left(\right.$ range $\left._{\mathrm{s}}\right)$ gives the number of distinct values of $\mathrm{k}_{\mathrm{s}}$ objects falling into the range ${ }_{\text {join }}$. The probability, $\mathrm{P}_{\mathrm{i}}$, that these two distinct values intersect is given as follows:

$$
P_{i}=o\left(\text { len }\left(\text { range }_{\text {join }}\right), \text { Dist }_{p}, \text { Dist }_{s}\right)
$$

The selectivity of the value-based two-path join, $\mathrm{q}^{\prime}$, is defined as the probality of the value of an object being in the intersection of two sets with cardinalities $\mathrm{k}_{\mathrm{p}}{ }^{*}$ len $\left(\right.$ range $\left._{j o i n}\right) /$ len $\left(\right.$ range $\left._{\mathrm{p}}\right)$ and $\mathrm{k}_{\mathrm{s}}{ }^{* *} \operatorname{len}\left(\right.$ range $\left._{\mathrm{join}}\right) /$ len(range ${ }_{\mathrm{s}}$ ) selected from a set whose cardinality is len(range join ) with replacement and it is calculated as:

$$
\mathrm{q}^{\prime}=\mathrm{P}_{\mathrm{i}} *\left(\mathrm{k}_{\mathrm{p}}^{\prime *} \operatorname{len}\left(\text { range }_{\text {join }}\right) / \operatorname{len}\left(\text { range }_{\mathrm{p}}\right)\right) *\left(\mathrm{k}_{\mathrm{s}}^{\prime *} \operatorname{len}\left(\text { range }_{\text {join }}\right) / \operatorname{len}\left(\text { range }_{\mathrm{s}}\right)\right) / \operatorname{len}\left(\text { range }_{\mathrm{join}}\right) * * 2
$$

Then, estimated join size is

$$
\text { join_size }=\operatorname{fref}\left(\mathrm{p} \cdot \mathrm{~A}_{1} \cdot \mathrm{~A}_{2} \ldots \mathrm{~A}_{\mathrm{m}}, \mathrm{t}_{\mathrm{s}}\right) * \operatorname{fref}\left(\mathrm{~s} \cdot \mathrm{~B}_{1} \cdot \mathrm{~B}_{2} \ldots \mathrm{~B}_{\mathrm{n}}, \mathrm{t}_{\mathrm{p}}\right) * \mathrm{q}^{\prime} .
$$

Case 3. Non-equality Joins
For such joins, estimated join size is taken as the cardinality of the cartesian product of two collections obtained by traversing the links in the path expressions, which is the result of a pessimistic assumption.

$$
\text { join size }=\operatorname{fref}\left(\mathrm{p} \cdot \mathrm{~A}_{1} \cdot \mathrm{~A}_{2} \ldots \mathrm{~A}_{\mathrm{m}}, \mathrm{t}_{\mathrm{s}}\right) * \operatorname{fref}\left(\mathrm{~s} \cdot \mathrm{~B}_{1} \cdot \mathrm{~B}_{2} \ldots \mathrm{~B}_{\mathrm{n}}, \mathrm{t}_{\mathrm{p}}\right)
$$

## 3. Cost Analysis of Operators in a Paging Environment

In this section a number of parameters will be defined which are used in the analysis of the costs of forward and backward traversals. These parameters are the costs of sequential, random and indexed accesses.

In Table 2 the physical parameters of a disk which are used in the numerical examples are provided.

| Parameter | Definition |
| :--- | :--- |
| B | Block size |
| btt | Block transfer time |
| ebt | Effective block transfer time |
| r | Avarege rotational latency |
| s | Avarage seek time |

Table 2. Physical Parameters for hard disk

The cost of sequential accesses to $b$ pages is denoted by $\operatorname{SEQCOST}(b)$ and is calculated as

$$
\operatorname{SEQCOST}(b)=s+r+b * e b t
$$

The cost of random access to $b$ pages, denoted by $\operatorname{RNDCOST}(b)$ is

$$
R N D C O S T(b)=b *(s+r+b t t) .
$$

In Table 3, the information kept by the system for a $\mathrm{B}^{+}$-tree index I is shown.

| Parameter | Definition |
| :--- | :--- |
| $\mathrm{v}(\mathrm{I})$ | Order of the $\mathrm{B}^{+}$tree |
| level(I) | Number of levels |
| leaves(I) | Number of pages at <br> the leaf level |
| keysize(I) | Size of the key value |
| unique(I) | Unique flag |

Table 3. Parameters for a $B^{+}$-tree
The cost of accessing object identifiers of $k$ random keys by using a $\mathrm{B}^{+}$tree secondary index $I$ is calculated as follows:

The number of pages at the leaf level of the $\mathrm{B}^{+}$tree is given as leaves(I). Assuming there are 2 v keys at each page and also assuming that the leaves are $\ln 2$ full [Yao 78], the total number of keys at the leaf level is leaves(I) $* 2 \mathrm{v} * \ln 2$. Then number of leaf level pages to be fetched in order to retrieve k keys is given by:

$$
\mathrm{c}\left(\mathrm{n}_{1}, \mathrm{~m}_{1}, \mathrm{r}_{1}\right)=\mathrm{c}(\operatorname{leaves}(\mathrm{I}) * 2 \mathrm{v} * \ln 2, \text { leaves }(\mathrm{I}), \mathrm{k})
$$

In other words, there are leaves(I)*2v*ln2 keys distributed over leaves(I) pages, and the question is, how many pages should we retrieve in order to get k keys from the leaf level of the $\mathrm{B}^{+}$tree index. Generalizing this to the upper levels of the secondary $\mathrm{B}^{+}$tree we obtain the formula, $\operatorname{INDCOST}(k)$, giving the cost of indexed access in terms of pages to be retrieved.

$$
\operatorname{INDCOST}(k)=\left(\sum_{i=1}^{\operatorname{level}(I)} c\left(n_{i}, m_{i}, r_{i}\right)\right) * R N D C O S T(1)
$$

where $\mathrm{n}_{\mathrm{i}}=\operatorname{leaves}(\mathrm{I}) /\left(2 \mathrm{v}^{*} \ln 2\right)^{\mathrm{i}-2}, \mathrm{~m}_{\mathrm{i}}=\operatorname{leaves}(\mathrm{I}) /\left(2 \mathrm{v}^{*} \ln 2\right)^{\mathrm{i}-1}$, and

$$
\begin{aligned}
& \mathrm{r}_{\mathrm{i}}=\quad \mathrm{k} \quad, \mathrm{i}=1 \\
& \mathrm{c}\left(\mathrm{n}_{\mathrm{i}-1}, \mathrm{~m}_{\mathrm{i}-1}, \mathrm{r}_{\mathrm{i}-1}\right), \text { otherwise. }
\end{aligned}
$$

Finally, the cost of a range query using a secondary $\mathrm{B}^{+}$-tree index $I$, given by $\operatorname{RNGXCOST}($ fract $)$, is given by the equation

$$
R N G X C O S T(\text { fract })=\text { fract } * \operatorname{leaves}(I) *(s+r+\text { btt })
$$

where fract is the proportion of the objects in the given range to the whole domain.

### 3.1 Cost of Forward Traversal

Assume that there exists a path expression, p. $\mathrm{A}_{1} . \mathrm{A}_{2} \ldots \mathrm{~A}_{\mathrm{m}}$, where $\mathrm{A}_{1}$ through $\mathrm{A}_{\mathrm{m}}$ are constructed using set, list or reference contructors, $A_{i}$ is an attribute of $C_{i}$ and $A_{m}$ references class $C_{m+1}$. We will calculate the cost of forward traversal for this expression. We use the short-hand notation for fan $_{i}$, totref ${ }_{i}$ and totlinks ${ }_{i}$ given previously.

Given $\mathrm{k}_{1}$ object identifiers of instances of $\mathrm{C}_{1}$, it is necessary to find out the cost of forward traversing a given path. In this analysis, the following assumptions are made:
i. No pages of an instance of $\mathrm{C}_{\mathrm{i}}$ is in the buffer, where $\mathrm{i}>1$.
ii. To eliminate the complexity of taking the effect of the page replacement policy of the buffer management system into account, the available buffer space is assumed to be large enough to hold all of the pages fetched during the traversal.

Let $k_{i}$ be the number of distinct instances of $C_{i}$ which are referenced by attribute $A_{i-1}$ of $C_{i-1}$. Then, after following the links from $\mathrm{C}_{1}$ to $\mathrm{C}_{2}$ for $\mathrm{k}_{1}$ objects in $\mathrm{C}_{1}$, the number of objects in $\mathrm{C}_{2}$, is given by $k_{2}$ :

$$
k_{2}=c\left(\text { totlinks }_{l}\left(A_{l}\right), \operatorname{totref}_{l}\left(A_{l}\right), k_{l} * \operatorname{fan}_{l}(A)\right)
$$

If there is a binary join index between the classes $C_{1}$ and $C_{2}$, which is implemented as a pair of $B+$ trees, through attribute $A_{1}$ then the cost of forward traversing into these $k_{2}$ items is just the cost of dereferencing them since the contents of $\mathrm{k}_{1}$ objects from $\mathrm{C}_{1}$ will not be accessed. Then, the cost function, bjicost, is written as:

$$
\text { bjicost }=\operatorname{INDCOST}\left(k_{l}\right)
$$

If there are no binary join indices, on the other hand, the cost is the total cost of accessing the pages over which $\mathrm{k}_{1}$ objects of $\mathrm{C}_{1}$ are distributed, together with the cost of accessing the blocks in which set information on object identifiers are stored, if $\mathrm{A}_{1}$ is a set. In this case, the cost is given as:

$$
\text { rcost }=\text { RNDCOST(nbpg) }+k_{1} * \operatorname{SETINFO}(I)
$$

where

$$
\text { nbpg }=\operatorname{nbpages}\left(C_{1}\right) *\left(1-\left(1-1 / \operatorname{nbpages}\left(C_{1}\right)\right)^{k l}\right) .
$$

and nbpg gives the number of pages to be fetched from class $C_{1}$ to retrieve $k_{1}$ objects, where $\left|C_{1}\right|$ objects are distributed over nbpages $\left(C_{1}\right)$ [Car 75].

If the type constructor of attribute $\mathrm{A}_{1}$ is of REFERENCE type, then obviously the second term of the equation for rcost, i.e. $k_{l} * \operatorname{SETINFO}(I)$, disappears. The parameter $\operatorname{SETINFO}(I)$ is a value proportional to the average cost of accessing the elements of a set organized as a $\mathrm{B}^{+}$-tree and it is as given below:

$$
\operatorname{SETINFO}(I)=(\operatorname{level}(I)+\operatorname{leaves}(I)) * R N D C O S T(1)
$$

The minimum of bjicost and rcost is chosen and added to the total forward traversal cost. Then the procedure for calculating the cost of a forward traversal of a path expression is as given below:

```
function ftracost( int k
{
    /* }\mp@subsup{\textrm{k}}{1}{}\mathrm{ is the initial number of object identifers of instances of class C}\mp@subsup{\textrm{C}}{1}{*}*
function ftracost
    int i = 1;
    float fcost=0, bjicost, rcost;
    while( i m m) {
        If (there are forward binary join indices on }\mp@subsup{\textrm{A}}{\textrm{i}}{})\mathrm{ ) bjicost = INDCOST( }\mp@subsup{\textrm{k}}{\textrm{i}}{})\mathrm{ ;
        else bjicost = \infty;
        nbpg = nbpages( }\mp@subsup{\textrm{C}}{\textrm{i}}{\textrm{i}})*(1-(1-1/nbpages(\mp@subsup{\textrm{C}}{\textrm{i}}{})\mp@subsup{)}{}{\textrm{ki}})
        switch( the constructor of }\mp@subsup{A}{i}{})\mathrm{ {
            case SET :
                rcost = RNDCOST(nbpg) + k k * ( SETINFO(I) );
            case REF :
                rcost = RNDCOST(nbpg);
    } /* switch */
    fcost = fcost + min(bjicost,rcost);
    i = i + 1;
    k
    } /* while */
    return(fcost);
} /* function */
```


## 3. 2 Cost of Backward Traversal

In our object model, there are no backward links. If there is binary join index between $\mathrm{C}_{\mathrm{i}}$ and $\mathrm{C}_{\mathrm{i}+1}$, the costs are calculated as in the forward traversal. However, if there are no binary join indices, instances in $\mathrm{C}_{\mathrm{i}}$ are selected by sequential scan of $\mathrm{C}_{\mathrm{i}}$. The number of instances, $\mathrm{k}_{\mathrm{i}}$, selected in $\mathrm{C}_{\mathrm{i}}$ as the result of this operation is estimated as:
$\mathrm{k}_{\mathrm{i}}=\left|\mathrm{C}_{\mathrm{i}}\right| *$ notnull $\left(\mathrm{A}_{\mathrm{i}}, \mathrm{C}_{\mathrm{i}}\right) * \mathrm{o}$ (totref $\mathrm{f}_{\mathrm{i}}$, fan $\left._{\mathrm{i}}, \mathrm{k}_{\mathrm{i}+1}\right)$, where $\mathrm{k}_{\mathrm{i}+1}$ is the number of instances in $\mathrm{C}_{\mathrm{i}+1}$ selected in the backward traversal of the path expression and o(totref $\mathrm{f}_{\mathrm{i}}, \mathrm{fan}_{\mathrm{i}}, \mathrm{k}_{\mathrm{i}+1}$ ) is the probability is that an object in $\mathrm{C}_{\mathrm{i}}$ gives reference to an instance in this set.

Given a path p. $\mathrm{A}_{1} \cdot \mathrm{~A}_{2} \ldots \mathrm{~A}_{\mathrm{m}}$, and keeping the assumptions made in the previous section, an analoguous algorithm for calculating the cost of backward traversal is given below.
function btracost( int $\mathrm{k}_{\mathrm{m}}$, PathExpression p. $\mathrm{A}_{1} \cdot \mathrm{~A}_{2} \ldots \mathrm{~A}_{\mathrm{m}}$ )
/* $\mathrm{k}_{\mathrm{m}}$ is the initial number of object identifers of instances of class $\mathrm{C}_{\mathrm{m}} * /$
int i;
float bcost=0, bjicost, scost;
$\mathrm{i}=\mathrm{m}$;
while ( $\mathrm{i} \neq 0$ ) \{
$\mathrm{k}_{\mathrm{i}}=\mathrm{k}_{\mathrm{i}} * \operatorname{hitprb}\left(\mathrm{~A}_{\mathrm{i}-1}, \mathrm{C}_{\mathrm{i}-1}, \mathrm{C}_{\mathrm{i}}\right)$;
bjicost $=\infty$;
If (there are binary join indices on $\mathrm{A}_{\mathrm{i}}$ )
bjicost $=\operatorname{INDCOST}\left(\mathrm{k}_{\mathrm{i}}\right)$;
scost $=\operatorname{SEQCOST}\left(\right.$ nbpages $\left.\left(\mathrm{C}_{\mathrm{i}}\right)\right)$;
bcost $=$ bcost $+\min ($ bjicost,scost $)$;
$\mathrm{i}=\mathrm{i}-1$;
$\mathrm{k}_{\mathrm{i}}=\left|\mathrm{C}_{\mathrm{i}}\right| * \operatorname{notnull}\left(\mathrm{~A}_{\mathrm{i}}, \mathrm{C}_{\mathrm{i}}\right) * \mathrm{o}\left(\right.$ totref $\left._{\mathrm{i}}, \operatorname{fan}_{\mathrm{i}}, \mathrm{k}_{\mathrm{i}+1}\right) ;$
\} /* while */
return(bcost);
\} $/ *$ function $* /$

## 4. An Example

In order to provide a better understanding to the use of the cost formulas introduced in the previous sections, the following example is provided.

Assume the following SQL query:
SELECT c.name
FROM Company c
WHERE c.division.staff.drives.color='blue'
and c.location.country= 'USA'

The physical parameters values of a typical disk drive is given in the table 4.

| Parameter | Value |
| :--- | :--- |
| B | 2 KByte |
| btt | 0.8 msec |
| ebt | 0.84 msec |
| r | 8 msec |
| s | 16 msec |

Table 4. Physical Parameters values for a typical hard disk
Let us also assume the database statistics are as shown in Table 5, Table 6 and Table 7.

| Class | $\|\mathrm{C}\|$ | nbpages(C) | size(C) |
| :--- | :--- | :--- | :--- |
| Company | 30 | 30 | 2 KByte |
| Division | 50 | 50 | 2 KByte |
| Employee | 15000 | 45000 | 6 KByte |
| Vehicle | 60000 | 150000 | 6 KByte |
| City | 30 | 30 | 2 KByte |

Table 5. Statistics on the example database

| Class | Attribute | dist |
| :--- | :--- | :--- |
| Vehicle | color | 10 |
| City | country | 10 |

Table 6. Statistics on the example database

| Class | Attribute | fan | totref | totlinks | hitprb |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Company | division | 4 | 40 | 120 | 0.80 |
| Division | staff | 150 | 7000 | 7500 | 0.47 |
| Employee | drives | 4 | 50000 | 60000 | 0.83 |
| Company | location | 1 | 30 | 30 | 1.00 |

Table 7. Statistics on the example database

The sequential scan costs of the class extensions are as given in Table 8.

| Class | Sequential Scan Cost(secs.) |
| :--- | :--- |
| Company | 0.0495 |
| Division | 0.0663 |
| Employee | 37.8243 |
| Vehicle | 126.0243 |
| City | 0.0495 |

Table 8. Sequential scan costs of the class extensions
The selectivities of the path expressions are calculated by the following formulas:

$$
\begin{aligned}
& f_{s}(P 1)=f_{s}(\text { c.division.staff.drives.color, }=)=1.00 \\
& f_{s}(P 2)=f_{s}(\text { c.location.country },=)=0.1
\end{aligned}
$$

By using the cost formula for backward traversal, btracost, the costs of the backward traversals of the path expressions P1and P2 are calculated to be:

$$
b_{P 1}=\text { Sequential scan cost of Vehicle to evaluate "color= 'blue’ " }+
$$ backward traversal of c.division.staff.drives

$=126.0243+\operatorname{btracost}\left(\mathrm{f}_{\mathrm{s}}(\text { color, }=)^{*} \mid\right.$ Vehicle $\mid$, c.division.staff.drives $)$
$=126.0243+37.9401$
$=163.9644 \mathrm{secs}$

$$
\begin{aligned}
& \mathrm{b}_{\mathrm{P} 2}=\text { Sequential scan cost of City to evaluate "country }= \text { 'USA' " }+ \\
& \quad \text { backward traversal of c.location } \\
&=0.0495+\text { btracost }\left(\mathrm{f}_{\mathrm{s}}(\text { country },=) * \mid \text { City } \mid, \text { c.location }\right) \\
&=0.0495+0.0495 \\
&=0.099 \mathrm{secs}
\end{aligned}
$$

It should be noted that there can be some fast access paths defined on the classes like access support relations [Kem 90] which are redundant seperate structures to store object references that are frequently traversed or path indices [Kim 90, Ber 93] which store instantiations of a path, that is, sequences of objects. We will assume that there are no such indices on the classes. Then, some of the available alternatives to be checked by the query optimizer are as follows:

1. A backward traversal of the predicate P1 followed by a forward traversal of the path expression P2.

The cost of this alternative is computed to be:

$$
\begin{aligned}
\mathrm{c} 1 & =\mathrm{b}_{\mathrm{P} 1}+\mathrm{ftracost}\left(\mathrm{f}_{\mathrm{s}}(\mathrm{P} 1) * \mid \text { Company } \mid, \text { c.location }\right) \\
& =163.9644+0.9628 \\
& =164.9272 \text { secs }
\end{aligned}
$$

2. A backward traversal of the path expression P2 followed by a forward traversal of the path expression P1. The cost of the second alternative is expressed as follows:

$$
\begin{aligned}
\mathrm{c} 2 & =\mathrm{b}_{\mathrm{P} 2}+\mathrm{ftracost}\left(\mathrm{f}_{\mathrm{s}}(\mathrm{P} 2) * \mid \text { Company } \mid, \text { c.division.staff.drives }\right) \\
& =0.099+119.024 \\
& =119.123 \text { secs. }
\end{aligned}
$$

3. Converting all implicit joins in the path expressions to the explicit joins, and executing these joins by using a pointer-based join algortihms like hybrid hash partitioning join or sort-merge join[She 90]. No matter what join technique is used, it will require at least one sequential scan of the file. Therefore the minimum cost for this alternative is greater than the sum of sequential scans of the extensions of the classes.

$$
\mathrm{c} 3>=163.9644 \text { secs } .
$$

It is clear from this example that for the optimizer to decide on the best execution plan among the available alternatives, it is in need of the selectivities and the costs of path expressions.

## 4. Conclusions

The query optimizer needs to know the cost of possible alternatives to choose the best one. The selectivities of attributes and the selectivities and the cost of explicit joins for relational data model are well-known. These formulas are also used for object-oriented query processing. However path
traversals, that is, executing precomputed joins is one of the possible alternatives in executing object-oriented queries. In this paper we have provided the formulas for calculating the selectivities of path expressions, for estimating size of an explicit join involving path expressions, and also for calculating the cost of forward and backward traversals.

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